

Grand unification in the minimal left-right symmetric extension of the standard model

Fabio Siringo

Dipartimento di Fisica e Astronomia, Università di Catania,

INFN Sezione di Catania and CNISM Sezione di Catania,

Via S.Sofia 64, I-95123 Catania, Italy

(Dated: August 20, 2012)

Abstract

The simplest minimal left-right symmetric extension of the standard model is studied in the high energy limit, and some consequences of the grand unification hypothesis are explored assuming that the parity breaking scale is the only relevant energy between the electro-weak scale and the unification point. While the model is shown to be compatible with the observed neutrino phenomenology, the parity breaking scale and the heavy boson masses are predicted to be above 10^7 TeV, quite far from the reach of nowadays experiments. Below that scale only an almost sterile right handed neutrino is allowed with a mass $M(\nu_R) \approx 100$ TeV.

PACS numbers: 12.60.Cn, 12.60.Fr, 12.10.Kt, 14.60.St

An interesting aspect in the phenomenology of neutrinos is the emerging of important elements of new physics beyond the standard model (SM), since there is no doubt that the experimental results can only be understood if the neutrinos are assumed to have nonvanishing masses and mixings. Massive neutrinos require the existence of a right-handed neutrino, which makes the B-L generator triangle anomaly free, and the related symmetry gaugeable. Thus the most natural extension of the SM gauge group is the Left-Right (LR) symmetric group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ which breaks to the SM group at some high scale[1–4]. LR models have been discussed as embedded in larger Grand Unification (GU) models like SO(10), and their symmetry breaking path has been discussed by several authors[5–8]. Recently [8–11] there has been a renewed interest in the minimal L-R symmetric extension of the SM[12, 13], a model that, with two scalar doublets and no bidoublets, predicts the low energy phenomenology of the SM with a very modest cost in terms of new particles that are required to be detected at very high energy.

Despite its simple particle content, the minimal model retains most of the interesting properties of more complex LR models: B-L is a gauge symmetry, with a triangle anomaly free generator; parity is spontaneously broken; massive neutrinos can be accommodated by seesaw mechanisms; dark matter could in part be accounted for by right handed neutrinos.

Even if at tree-level the minimal LR model, without bidoublets, fails to predict a stable broken symmetry vacuum[14], a consistent path for the breaking of symmetry has been predicted by inclusion of higher order corrections that become relevant in the Higgs-Top sector[15]. Other paths towards the breaking of symmetry have been recently discussed[9, 11], and the minimal model seems to be a viable first step towards new physics beyond the SM.

However, while there is a certain amount of results on the "standard" LR model[16–19], the more recent minimal LR model has not been studied enough. Like other non-susy models, there is evidence that an intermediate high scale is required before unification[20]. Thus it would be interesting to investigate the issue of high energy unification in the framework of the minimal LR model.

In this paper we present a detailed quantitative analysis of the most simple symmetry breaking path for the minimal LR model up to unification, in order to pinpoint its predictions for the breaking scales and neutrino masses. While similar analysis have been presented for more complex susy LR models[21, 22], we notice that the exact behaviour of gauge couplings

depends on the detailed particle content of the model, and it is interesting to address the question in a truly minimal LR model with a minimum particle content.

We show that the simple hypothesis that a single intermediate scale exists between GU and the weak scale, is enough for predicting this intermediate scale and the masses of the heavy gauge bosons Z' and W_R . The prediction of a breaking scale of order 10^{10} GeV, halfway between the electro-weak scale and the GU scale, is encouraging even if that scale seems to be too large to be detected by nowadays experiments. The results are compatible with a micro-milli-eV mass scenario for neutrinos, and show that the non-susy minimal LR model is a valid and natural option as a first step towards the understanding of new physics beyond the SM.

An interesting point is that the present analysis does not make any use of the details of the model above GU, and does not require full knowledge of the symmetry breaking mechanism nor detailed descriptions of the minimal set of Higgs representations: the beta functions only depend on the actual particle content of the model below unification. This generality makes the analysis valid for a quite large range of mechanisms and even for different unifying groups. On the other hand, this choice of generality can be regarded as a shortcoming of the present study, just because no answer can be given to important issues like the details on the emerging of the low energy Lagrangian, the flow of the merged couplings above GU, the proton lifetime prediction, and even the details of the unifying group. Nevertheless the analysis is very simple and its generality makes it worth to be discussed together with its possible effects on the physics of neutrinos.

The minimal LR symmetric model has been described in several papers[12–15]. The LR symmetric lagrangian is the sum of a fermionic term \mathcal{L}_f , a standard Yang-Mills term \mathcal{L}_{YM} for the gauge bosons, a Higgs term \mathcal{L}_H and eventually the Higgs-fermion interaction term \mathcal{L}_{int} . A special feature of the minimal model is its limited particle content. The Higgs sector contains two scalar doublets but no bidoublet, and is described by the simple Lagrangian

$$\mathcal{L}_H = -\frac{1}{2}|D_L^\mu \chi_L|^2 - \frac{1}{2}|D_R^\mu \chi_R|^2 + V(\chi_L, \chi_R) \quad (1)$$

where the covariant derivative D_a^μ is defined according to

$$D_a^\mu = \left(\partial^\mu - ig_a \vec{A}_a^\mu \vec{T}_a + i\tilde{g} B^\mu \frac{Y}{2} \right), \quad a = L, R. \quad (2)$$

\vec{T}_L , \vec{T}_R and Y are the generators of $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively, with couplings $g_L = g_R = g$ and \tilde{g} . The electric charge is given by $Q = T_{L3} + T_{R3} + Y/2$. The Higgs fields

χ_a are the scalar doublets

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}, \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} \quad (3)$$

with the transformation properties

$$\chi_L \equiv (2, 1, 1), \quad \chi_R \equiv (1, 2, 1). \quad (4)$$

A standard \mathcal{L}_{YM} is considered for the seven gauge fields \vec{A}_L^μ , \vec{A}_R^μ and B^μ .

Fermions are described by doublets of spinors ψ_L , ψ_R with the transformation properties

$$\psi_L \equiv (2, 1, B - L), \quad \psi_R \equiv (1, 2, B - L). \quad (5)$$

Their lagrangian term \mathcal{L}_f is

$$\mathcal{L}_f = -\bar{\psi}_L \gamma_\mu D_L^\mu \psi_L - \bar{\psi}_R \gamma_\mu D_R^\mu \psi_R \quad (6)$$

The lagrangian $\mathcal{L} = \mathcal{L}_f + \mathcal{L}_{YM} + \mathcal{L}_H$ is fully symmetric for L-R exchange if the Higgs potential $V(\chi_L, \chi_R)$ is assumed to be symmetric for the exchange of χ_L and χ_R .

The simplest path for symmetry breaking requires two energy scales[15]: parity is assumed to be broken at a large energy scale $\mu = \Lambda_R$ where the scalar R-doublet χ_R takes a broken symmetry vacuum expectation value (vev), $\langle \chi_R \rangle = w$, while the L-doublet χ_L still retains a vanishing vev. Below this energy scale the gauge group is broken to the SM gauge group $SU(2)_L \otimes U(1)$. At the electroweak scale the L-doublet χ_L takes a broken symmetry vev $\langle \chi_L \rangle = v$, breaking the SM gauge group to the simple $U(1)_{em}$ group of electromagnetism. Provided that $w \gg v = 246$ GeV the model predicts the same phenomenology of the SM. In unitarity gauge we set $\chi_a^+ = 0$ and take χ_a^0 real with a finite vev $\langle \chi_L^0 \rangle = v$, $\langle \chi_R^0 \rangle = w$. Assuming $v \ll w$, the mass matrix for the gauge bosons has two charged eigenvectors[12] W_L^\pm and W_R^\pm which are decoupled with masses

$$M_{W(L)} = \frac{gv}{2}, \quad M_{W(R)} = \frac{gw}{2}, \quad (7)$$

a vanishing eigenvalue for the electromagnetic unbroken $U(1)_{em}$ eigenvector, and two massive neutral eigenvectors with a small mass

$$M_Z^2 = \frac{g^2 v^2 (g^2 + 2\tilde{g}^2)}{4(g^2 + \tilde{g}^2)} + \mathcal{O}(v^2/w^2) \quad (8)$$

for the “light” Z boson, and a large mass

$$M_{Z'}^2 = (M_{W(L)}^2 + M_{W(R)}^2) (1 + \tilde{g}^2/g^2) - M_Z^2 \quad (9)$$

for the “heavy” boson Z' . Of course, at low energy, all the effects of the heavy Z' and W_R^\pm are suppressed[12].

In an intermediate energy range, above the electroweak scale up to the parity breaking scale Λ_R , the minimal LR model mimics the SM with a $SU(2)_L$ gauge coupling $g_2 = g$ and a $U(1)$ coupling g_1 that according to Eq.(8) must satisfy the matching condition

$$\frac{1}{g_1^2} = \frac{1}{\tilde{g}^2} + \frac{1}{g_2^2} \quad (10)$$

at the scale $\mu = \Lambda_R$, in order to recover the known SM result $2M_Z^2/v^2 = g_2^2 + g_1^2$.

The GU hypothesis of a single unified gauge symmetry describing all forces and matter at very short distances is very attractive and, according to it, the couplings are expected to merge at a very high energy scale $\mu = \Lambda_{GUT}$. However, as in other non-susy models, an intermediate high scale is required before unification[20]. In this paper we explore the simplest hypothesis that the intermediate scale is the parity breaking scale $\mu = \Lambda_R$, and that above that scale the gauge couplings of the full gauge group $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ run up to the unification scale $\mu = \Lambda_{GUT}$ where they merge. We show that this simple hypothesis is enough for determining the scales Λ_R and Λ_{GUT} , by a simple use of the known beta functions of the model.

We prefer the use of simple one-loop beta functions that are decoupled and allow for a full analytical discussion of the problem. Two-loop beta functions are known for the standard model[25] and would be required for a full quantitative discussion, but their use would not change the qualitative result in any way.

At one-loop the gauge couplings satisfy the renormalization group (RG) equation

$$\mu \frac{dg_i}{d\mu} = \beta_i(g_i) = -b_i \frac{g_i^3}{16\pi^2} \quad (11)$$

where the coefficient b_i is known to be[26]

$$b_i = \frac{11}{3}C_A - \frac{4}{3}(2n_g T_F) - \frac{1}{3}T_s n_s, \quad (12)$$

n_g is the number of fermion generations and n_s is the number of complex scalars.

For the gauge groups $SU(2)$ and $SU(3)$ the running of the couplings is not affected by the breaking of symmetry at the scale $\mu = \Lambda_R$, and the coefficients are $b_2 = 19/6$ for $SU(2)$ ($C_A = 2$, $T_F = 1/2$, $n_g = 3$, $T_s = 1/2$ and $n_s = 1$) and $b_3 = 7$ for $SU(3)$ ($C_A = 3$, $T_F = 1/2$, $n_g = 3$ and $n_s = 0$). In fact for both groups the trace of the square of a generator T reads

$$\text{Tr}(g_i T \cdot g_i T) = 4g_i^2 n_g T_F = 2g_i^2 n_g \quad (13)$$

For the $U(1)_{B-L}$ gauge group the running of the coupling depends on the particle content that is different below and above the breaking symmetry scale. For $\mu > \Lambda_R$ the LR symmetry is unbroken and for each generation there are six left-handed quarks with $Y = 1/6$, six right-handed quarks with $Y = 1/6$, two left handed leptons with $Y = 1/2$ and two right-handed leptons with $Y = 1/2$. Thus

$$\text{Tr}(\tilde{g}T \cdot \tilde{g}T) = \sum_{\text{fermions}} (\tilde{g}^2 Y^2) = \frac{4}{3} \tilde{g}^2 n_g \quad (14)$$

which is equivalent to set $T_F = 1/3$ in Eq.(12), and since there are two scalars, $n_s = 2$, we obtain the coefficient $\tilde{b} = -3$ for the coupling \tilde{g} of $U(1)_{B-L}$. For $\mu < \Lambda_R$ the LR symmetry is broken and the coupling g_1 is determined by the beta function of the SM $U(1)$ gauge group: for each generation there are six left-handed quarks with $Y = 1/6$, three right-handed quarks with $Y = 2/3$, three right-handed quarks with $Y = -1/3$, two left handed leptons with $Y = -1/2$ and one right-handed lepton with $Y = -1$. Thus

$$\text{Tr}(g_1 T \cdot g_1 T) = \sum_{\text{fermions}} (g_1^2 Y^2) = \frac{10}{3} g_1^2 n_g \quad (15)$$

which is equivalent to set $T_F = 5/6$ in Eq.(12), and since there is one scalar, $n_s = 1$ (the heavy fields are integrated out), we obtain the SM coefficient $b_1 = -41/6$ for the coupling g_1 .

It is useful to rescale the couplings in order to make explicit the equivalence of the trace in Eqs.(13),(14) and (15). Let us define the new set of couplings

$$\alpha_1 = \frac{5}{3} \frac{g_1^2}{4\pi}; \quad \tilde{\alpha} = \frac{2}{3} \frac{\tilde{g}^2}{4\pi} \quad (16)$$

$$\alpha_2 = \frac{g_2^2}{4\pi}; \quad \alpha_3 = \frac{g_3^2}{4\pi} \quad (17)$$

In fact the GU hypothesis requires that the trace in Eq.(13) and in Eq.(15) must be the same at the GU scale $\mu = \Lambda_{GUT}$ where $SU(3)$, $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ are restored as sub-groups of the same larger group. In terms of the new set of rescaled couplings the equivalence of the trace is satisfied whenever the couplings are equal, and the condition for GU is simply stated as $\tilde{\alpha} = \alpha_2 = \alpha_3$.

The new set of couplings satisfy the RG equation

$$\mu \frac{d\alpha_i}{d\mu} = \beta_i(\alpha_i) = -2c_i \frac{\alpha_i^2}{4\pi} \quad (18)$$

where $c_2 = b_2$, $c_3 = b_3$, $c_1 = 3b_1/5$ and $\tilde{c} = 3\tilde{b}/2$.

Eq.(18) can be easily integrated yielding the linear equations

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) + \frac{c_i}{2\pi} \ln \left(\frac{\mu}{\mu_0} \right) \quad (19)$$

that are reported in Fig.1. In this scenario the scale Λ_{GUT} is determined by the crossing of α_2 and α_3 . Inclusion of two-loop corrections would decrease the value of $\ln \Lambda_{GUT}$ by less than 3%[25], and would not affect the order of magnitude of Λ_{GUT} . Two-loop corrections are even smaller at the intermediate scale Λ_R and they are completely negligible at the electro-weak scale.

As discussed above, we assume that at the scale $\mu = \Lambda_{GUT}$ all the couplings cross, yielding $\tilde{\alpha}(\Lambda_{GUT}) = \alpha_2(\Lambda_{GUT}) = \alpha_3(\Lambda_{GUT})$. By the RG Eq.(19) the coupling $\tilde{\alpha}$ is let flow down to the parity breaking scale Λ_R where, according to the matching condition Eq.(10), must satisfy the constraint

$$\alpha_1(\Lambda_R) = \frac{5\alpha_2(\Lambda_R)\tilde{\alpha}(\Lambda_R)}{2\alpha_2(\Lambda_R) + 3\tilde{\alpha}(\Lambda_R)} \quad (20)$$

with $\alpha_1(\Lambda_R)$ that can be determined by the SM beta function Eq.(19) for $\mu < \Lambda_R$, starting from the known value at the electro-weak scale $\alpha_1(M_Z)$ and flowing up to the matching point Λ_R . The unknown scale Λ_R is pinpointed by the matching Eq.(20) as shown in Fig.1.

The analytical solution is

$$\ln \left(\frac{\Lambda_{GUT}}{M_Z} \right) = 2\pi \left(\frac{\alpha_2^{-1} - \alpha_3^{-1}}{c_3 - c_2} \right) = \frac{12\pi}{23} (\alpha_2^{-1} - \alpha_3^{-1}) \quad (21)$$

$$\ln \left(\frac{\Lambda_R}{M_Z} \right) = 2\pi (A\alpha_1^{-1} + B\alpha_2^{-1} + C\alpha_3^{-1}) \quad (22)$$

where $\alpha_i = \alpha_i(M_Z)$ are the couplings evaluated at the electro-weak scale $\mu = M_Z$, and

$$A = \left(\frac{3}{5}c_2 + \frac{2}{5}\tilde{c} - c_1 \right)^{-1} = \frac{5}{21} \quad (23)$$

$$B = \frac{2A}{5} \left(\frac{\tilde{c} - c_3}{c_3 - c_2} - \frac{3}{2} \right) = -\frac{3}{7} \quad (24)$$

$$C = \frac{2A}{5} \left(\frac{c_2 - \tilde{c}}{c_3 - c_2} \right) = \frac{4}{21}. \quad (25)$$

Inserting the actual phenomenological values[27] $\alpha_1^{-1} = 59.01$, $\alpha_2^{-1} = 29.57$, $\alpha_3^{-1} = 8.33$ in Eq.(22) we obtain a parity breaking scale

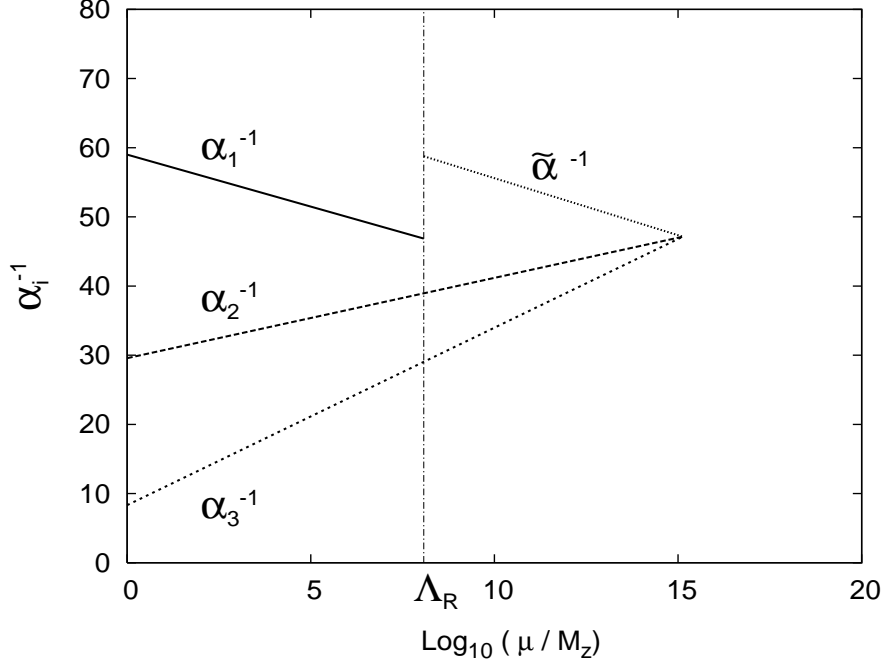


FIG. 1: Running of the inverse couplings α_1^{-1} , α_2^{-1} , α_3^{-1} for the SM $SU(3) \otimes SU(2)_L \otimes U(1)$ gauge group, below the parity breaking scale ($\mu < \Lambda_R$, on the left), and of the inverse couplings $\tilde{\alpha}^{-1}$, α_2^{-1} , α_3^{-1} for the LR symmetric $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group, above the parity breaking scale ($\mu > \Lambda_R$, on the right).

$$\frac{\Lambda_R}{M_Z} = 1.2 \cdot 10^8 \quad (26)$$

that is halfway between the electroweak scale and the GU scale that by Eq.(21) is predicted to be

$$\frac{\Lambda_{GUT}}{M_Z} = 1.3 \cdot 10^{15}. \quad (27)$$

A scale $\Lambda_R \approx 10^7$ TeV, while far from the reach of nowadays experiments, is in agreement with the predictions of other “standard” LR models[17–19]. It is quite reasonable to believe that the vev of the R-scalar χ_R is $w \approx \Lambda_R$ [15], and that $v/w \approx 10^{-8}$. At the LHC energy $\sqrt{s} = 14$ TeV the existence of tiny corrections of order $\sqrt{s}/w \approx 10^{-6}$ would be hardly detected, and once more a confirm of the present scenario could only come from the physics of neutrinos.

In the minimal LR model the mass generation can be understood in terms of non-renormalizable effective operators that are generated at low energy below the symmetry breaking scale. Mass terms can be generated by bilinear fermionic operators that must be

coupled with Higgs bidoublets or triplets respectively for Dirac or Majorana masses, in order to preserve gauge invariance. In the minimal model a Higgs bidoublet can be written as the product $\chi_L \chi_R^\dagger$ of a $SU(2)_L$ doublet times a $SU(2)_R$ doublet, yielding a factor vw in the low energy limit, and Dirac mass terms $m_D \bar{\psi}_L \psi_R = \gamma_D \bar{\psi}_L \psi_R vw$. A triplet can be built up from two $SU(2)_L$ doublets (or two $SU(2)_R$ doublets) yielding a factor v^2 (or w^2) in the low energy limit, and Majorana mass terms $M_L \bar{\psi}_L^C \psi_L = \gamma_M \bar{\psi}_L^C \psi_L v^2$, $M_R \bar{\psi}_R^C \psi_R = \gamma_M \bar{\psi}_R^C \psi_R w^2$. Here the couplings γ_D and γ_M are expected to scale like the inverse of some large energy scale Λ .

Thus for neutrinos the mass matrix can be written as

$$\begin{pmatrix} M_L & m_D \\ m_D & M_R \end{pmatrix} = m_D \begin{pmatrix} y \frac{v}{w} & 1 \\ 1 & y \frac{w}{v} \end{pmatrix} \quad (28)$$

where $y = \gamma_M/\gamma_D$ is of order unity, and the Dirac mass m_D is expected to fall in the MeV-GeV range like for other fermions. In fact for charged fermions $y = 0$ and the mass matrix contains only Dirac terms. In the present argument a single generation is considered for neutrinos. Of course the discussion of important aspects like mixing among light neutrinos would require a full mass matrix, but the existence of mixing terms would not change in any important way the qualitative nature of the argument. The eigenvalues of the mass matrix Eq.(28) show the usual seesaw behaviour with a light neutrino ν_L

$$M(\nu_L) = \frac{y^2 - 1}{2} \left(\frac{v}{w} \right) m_D + \mathcal{O} \left(\frac{v^2}{w^2} \right) \quad (29)$$

and a heavy neutrino ν_R

$$M(\nu_R) = y \left(\frac{w}{v} \right) m_D + \mathcal{O} \left(\frac{v}{w} \right). \quad (30)$$

Assuming that $v/w \approx M_Z/\Lambda_R \approx 10^{-8}$, the mass of the light neutrino ν_L would be pushed below the eV scale while a heavy neutrino ν_R would be conceivable with a mass $M(\nu_R) = m_D \cdot 10^8 \approx 10^8 \text{ MeV} = 100 \text{ TeV}$. At the LHC energy the ratio $\sqrt{s}/M(\nu_R) \approx 0.1$, but the heavy neutrino only interacts through the heavy gauge bosons W_R , Z' with an effective weak coupling that scales like $M_{W_L}^2/M_{W_R}^2 = v^2/w^2 \approx 10^{-16}$ compared to the light neutrino. Thus its sterile nature would prevent its detection anyway. Astrophysical effects could be considered, as the large mass of the heavy neutrino would imply important gravitational effects, and the dark matter of the universe could in part be accounted for by sterile neutrinos.

In summary, the simplest minimal LR extension of the SM has been studied in the high energy limit, and some consequences of the GU hypothesis have been explored assuming

that the parity breaking scale Λ_R is the only relevant energy above the electro-weak scale up to GU. In this scenario, which is shown to be compatible with the observed neutrino phenomenology, the parity breaking scale and the heavy boson masses are pushed up to 10^7 TeV, quite far from the reach of nowadays experiments. Below that scale only an almost sterile right handed neutrino could exist with a mass $M(\nu_R) \approx 100$ TeV.

-
- [1] J.C. Pati and A. Salam, Phys.Rev. D**10**, 275 (1974).
 - [2] R.N. Mohapatra and J.C. Pati, Phys.Rev.D **11**, 566 (1975).
 - [3] G. Senjanovic and R.N. Mohapatra, Phys.Rev.D **12**, 1502 (1975).
 - [4] G. Senjanovic, Nucl.Phys.B **153**, 334 (1979).
 - [5] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993).
 - [6] D. Caldwell and R. N. Mohapatra, Phys. Rev. D **48**, 3259 (1993); Phys.Rev.D **50**, 3477 (1994).
 - [7] R. N. Mohapatra, UMD-PP-94-160, arXiv:hep-ph/9405358v1
 - [8] J.N. Esteves, J.C. Romao, M. Hirsch, W. Porod, F. Staub, A. Vicente, JHEP 1201, 095 (2012).
 - [9] P.H. Gu, Phys. Rev. D **81**, 095002 (2010).
 - [10] P.H. Gu and M. Lindner, Phys. Lett. B **698**, 40-43 (2011).
 - [11] H. Chavez, J. A. Martins Simoes, Eur. Phys. J. C **50**, 85 (2007).
 - [12] F. Siringo, Eur. Phys. J. C **32**, 555 (2004); hep-ph/0307320.
 - [13] B. Brahmachari, E. Ma, U. Sarkar, Phys. Rev. Lett. **91**, 011801 (2003).
 - [14] F. Siringo, Phys. Rev. Lett. **92**, 119101 (2004).
 - [15] F. Siringo and L. Marotta, Phys. Rev. D **74**, 115001 (2006)
 - [16] R. N. Mohapatra, *Unification and Supersymmetry*, GTP, Springer (2003).
 - [17] D.Chang, R.N.Mohapatra, J.Gipson, R.E.Marshak and M.K.Parida, Phys. Rev. D**31**, 1718 (1985).
 - [18] R. N. Mohapatra and M. K. Parida, Phys. Rev. D **47**, 264 (1993).
 - [19] N. G. Deshpande, R. Keith and P. B. Pal, Phys. Rev. D **46**, 2261 (1992).
 - [20] A. Melfo, A. Ramirez, G. Senjanovic, Phys. Rev. D **82**, 075014 (2010).
 - [21] S. Patra, A. Sarkar, U. Sarkar, Phys. Rev. D **82**, 015010 (2010).
 - [22] D. Borah, S. Patra, U. Sarkar, Phys. Rev. D **83**, 035007 (2011).
 - [23] That left-right symmetric theories predict the standard model phenomenology has been

shown on very general grounds by Senjanovic[4] who employed the method of Georgi and Weinberg[24].

- [24] H. Georgi and S. Weinberg, Phys.Rev.D **17**, 275 (1978).
- [25] H. Arason, D.J. Castano, B. Kesthelyi, S. Mikaelian, E.J. Piard, P. Ramond, B.D. Wright, Phys. Rev. D **46**, 3945 (1992).
- [26] J.Y. Chiu, F. Golf, R. Kelley, A. V. Manohar, Phys. Rev. D **77**, 053004 (2008).
- [27] K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010).